CHANNEL CROSS-MIXING IN A FINNED-ROD BUNDLE

B. V. Dzyubenko

UDC 536.242:621.039.534

A method is given for processing data on channel cross-mixing in bundles of finned rods, which allows one to analyze various sources of data and to derive general relationships for the mixing coefficients.

Bundles of cylindrical rods or tubes with spiral fins or wire packing are widely used in power plants of various types [1-3]. These rods accelerate the channel cross-mixing in a bundle, which evens out nonuniformities in the temperature and velocity patterns, since these may arise from uneven energy production and other causes [4].

There are two ways of calculating the temperature distribution in such a rod bundle. For example, the exchange rates between coolant flows in adjacent cells in a bundle may be characterized by a mixing coefficient, which is the proportion of the flow exchanged per unit length taken relative to the total flow in a channel [4]:

$$\bar{\mu} = G_{ij}/G_i. \tag{1}$$

Calculations on the temperature patterns involve solving a system of energy equations written for the cells and closed by means of observed channel cross-mising coefficients $\bar{\mu}$ [4].

If the system has a large number of rods, one can also use an approach involving homogenization of the real bundle [5]. The flow of the homogenized medium is then described by the equations for a continuous medium. The homogenization effect is incorporated by the factor $(1 - m)/m$, where m is the porosity of the rod bundle for the coolant with the allowance for the thickness of the boundary layer, the condition being that the heat balance is maintained:

$$\int_{T_{\text{in}}}^{T_{\text{out}}} Gc_{\text{p}}dT = Q. \tag{2}$$

For the purposes of the treatment we assume that the rods and housing bear a layer of material equal in thickness to the boundary layer δ^* [6], and then we consider the free flow with slip of the homogenized medium in the new boundaries with distributed sources of bulk energy production and hydraulic resistance. This model gives the following system of differential equations for the axisymmetrical case for turbulent motion in a bundle of rods on the basis that the velocity vector is everywhere parallel to the axis of the bundle:

$$\rho u \frac{\partial u}{\partial x} = -\frac{dP}{dx} + \frac{1}{r}\frac{\partial}{\partial r}\left(\rho r D_t \Pr_{\text{T}} \frac{\partial u}{\partial r}\right) - \xi \frac{\rho u^2}{2d_{\text{e}}}, \tag{3}$$

$$\rho u c_{\text{p}} \frac{\partial T}{\partial x} = q_V \frac{1-m}{m} + \frac{1}{r}\frac{\partial}{\partial r}\left(\rho r c_{\text{p}} D_t \frac{\partial T}{\partial r}\right), \tag{4}$$

$$G = 2\pi m \int_0^{R_0} \rho u r dr, \tag{5}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 37, No. 5, pp. 777-783, November, 1979. Original article submitted January 23, 1979.

 0022-0841/79/3705-1262$07.50

$$\rho = \rho(P,\ T),\ \mu = \mu(P,\ T),\ \frac{\lambda}{c_p} = \frac{\lambda}{c_p}(P,\ T). \tag{6}$$

The boundary conditions are

$$u(0,\ r) = u_{\rm in},\ T(0,\ r) = T_{\rm in},\ P(0,\ r) = P_{\rm in}, \tag{7}$$

$$\left.\frac{\partial u(x,\ r)}{\partial r}\right|_{r=R_0} = 0,\quad -\lambda\left.\frac{\partial T(x,\ r)}{\partial r}\right|_{r=R_0} = 0, \tag{8}$$

$$\left.\frac{\partial u(x,\ r)}{\partial r}\right|_{r=0} = 0,\quad \left.\frac{\partial T(x,\ r)}{\partial r}\right|_{r=0} = 0. \tag{9}$$

This system may be solved by numerical methods. An analogous system for the three-dimensional case has been solved by matrix factorization [7, 8], so the main purpose of this study is not to solve (3)-(6), but to define a general relationship for the effective diffusion coefficient D_t in relation to the parameters that determine the mixing in bundles of finned rods with turbulent flow. The value of D_t has to be known in order to close (3)-(6). The need for this arises because at present no formulas are available for D_t.

The value of D_t is uniquely related to the mixing factor $\bar{\mu}$, which can be shown as follows. The amount of heat Q_{ij} transferred from cell i to cell j in unit length is

$$Q_{ij} = G_{ij}c_p(T_i - T_j),\quad Q_{ij} = \rho h D_t c_p\left(\frac{\partial T}{\partial y}\right)_{ij}, \tag{10), (11}$$

where $h = p - d_r$ and p is the pitch of the rod lattice.

We equate the right sides of (10) and (11) to get for $(\partial T/\partial y)_{ij} = (T_i - T_j)/y_{ij},\ y_{ij} = h$ that

$$G_{ij} = \rho D_t. \tag{12}$$

We divide the left and right parts of (12) by

$$G_i = \rho u F_c, \tag{13}$$

to get

$$G_{ij}/G_i = D_t/uF_c. \tag{14}$$

We introduce the dimensionless effective diffusion coefficient

$$\bar{k} = D_t/ud_e \tag{15}$$

Then from (14) with $F_c \approx mp^2/2 \approx 0.5p^2/2$ we get

$$\bar{k} = \bar{\mu}p^2/4d_e \tag{16}$$

The value of $\bar{k}$ defined by (15) is used to close (3)-(6) and in the present case is constant over the flow region; the effective diffusion coefficient allows for the heat transfer by turbulent diffusion, secondary circulation in the cells, and convective transfer in the cross section of the bundle on the scale of the diameter. These transfer mechanisms are incorporated by $\bar{\mu}$.

From (16) we can calculate the dimensionless effective diffusion coefficient $\bar{k}(D_t)$ from experimental data on $\bar{\mu}$ from various sources [2-4].

To relate $\bar{k}$ to the definitive criteria we use the π theorem in the theory of dimensions and the hydraulic-diameter rule, which provides for approximate similarity in bundles of finned rods. The equivalent diameter is

$$d_e = 4F_u/\Pi_w \tag{17}$$

We can use the π theorem in combination with the following physical arguments for geometrically dissimilar rod bundles. When the coolant flows in the spiral channel the liquid is subject to centrifugal forces, which produce secondary circulation in the cross section, which expands the core of the flow and accelerates the heat transport in the transverse direction. If we use the acceleration g_{cm} as the parameter characterizing the centrifugal forces, we get the following system of parameters characterizing the flow of an incompressible liquid over the stabilized section in a bundle of finned rods under nearly isothermal condtions: d_e, ρ, μ, u_{av}, and g_{cm}. This system is used with the three independent units of measurement (m, kg, and sec) to get from the π theorem [9] that there are two dimensionless combinations:

$$\mathrm{Re} = u_{av} d_e \rho/\mu, \tag{18}$$

$$\mathrm{Fr}_c = u_{av}^2/g_{cm} d_e. \tag{19}$$

If we assume that the coolant flow in the spiral channels follows the law

$$u_\tau/r = \mathrm{const}, \tag{20}$$

then the maximum value of the tangential velocity component u_τ is given by

$$u_{\tau m} = \pi d_r u_{av}/S, \tag{21}$$

while the acceleration is given by

$$g_{cm} = 2u_{\tau м}^2/d_r . \tag{22}$$

We substitute (21) into (22) and (22) into (19) to get

$$\mathrm{Fr}_c = S^2/2\pi^2 d_r d_e. \tag{23}$$

Instead of Fr_c we can use the modified quantity

$$\mathrm{Fr}_m = S^2/d_r d_e. \tag{24}$$

Thus numbers (18) and (24) characterize the flow in a bundle of finned cylindrical rods and allow one to generalize from the experimental data on the mixing coefficients for geometrically dissimilar rod bundles and thus get a formula for $\bar{k}$.

We determine $\bar{k}$ from the experimental data on $\bar{\mu}$ of [2, 3], which were obtained by heating the central rod; these studies were based on assemblies of 61 rods with single-start wire winding to produce a rib on the body, while the coolants were sodium and air. The geometrical dimensions of the bundles of cylindrical rods in [2, 3] are given in Table 1 along with the observed values of $\bar{\mu}$.

It is found [2, 3] that the Reynolds number does not affect $\bar{\mu}$ in the range Re = $0.8 \cdot 10^4 - 7 \cdot 10^4$ covered by the experiment; therefore, $\bar{\mu}$ and $\bar{k}$ are affected only by Fr_m together with the transformed longitudinal coordinate $2ax/d_r$ if the length of the bundle is short. To determine $\bar{k}$ we compare the measured temperature patterns with the calculated ones from (3)-(6) on the assumption that $\bar{k}$ is constant along the rod bundle. However, if only a single rod is heated, there is in fact a fairly extended initial part on which a universal temperature profile is built up. The value of $\bar{k}$ may be taken as constant, and therefore $\bar{\mu}$ also, only for the stabilized flow section, or else as approximately constant over the entire length if this is large enough, when the effects of the initial part may be neglected. These values of $\bar{k}$ and $\bar{\mu}$ are called asymptotic and denoted, respectively, by $\bar{k}_{as}$, $\bar{\mu}_{as}$; the length of the initial part increases with Fr_m.

The dependence of $\bar{k}_{as}$ ($\bar{\mu}_{as}$) on Fr_m can be defined as follows. We introduce the transformed longitudinal coordinate in the bundle $2ax/d_r$, where a is a coefficient representing the structure [10], which characterizes the rate at which a jet dies out when it propagates in the bundle, which is independent of Fr_m (of the pitch of the fins). We assume that the jet in the bundle propagates in accordance with laws close to those for an enclosed jet, while the angle of expansion of the jet is twice the angle of the spiral winding. Then we have the following relationship for a:

$$a = 0.075 + \frac{11{,}37}{\mathrm{Fr}_m} + \frac{246}{\mathrm{Fr}_m^2}, \tag{25}$$

TABLE 1. Initial Geometrical Dimensions of Finned-Rod Bundles and Experimental Data on the Mixing Coefficient (bundles Nos. 1-3 [2], No. 4 [3])

Parameter	Bundle No.			
	1	2	3	4
Coolant	Sodium	Sodium	Sodium	Air
Pitch S of winding, mm	100	200	300	100
Diam. of cylindrical rod, mm	6	6	6	6
Diam d_r of rod with winding, mm	7,9	7,9	7,9	7,02
Ratio S/d_r	12,7	25,3	38	14,2
Equiv. diam. d_e, mm	4	4	4	2,633
	25	50	75	38
Ratio S/d_e	0,38	0,38	0,38	—
Fr_m	318	1260	2850	540
Length of rod bundle, mm	1000	1000	1000	660
Coefficients:				
μ, 1/cm	0,11	0,0535	0,032	0,067
μ_{min}, 1/cm	0,107	0,047	0,027	0,057
μ_{max}, 1/cm	0,113	0,06	0,037	0,077
$\overline{k}$	0,043	0,0208	0,01245	0,0312
k_{min}	0,0418	0,0183	0,0105	0,0266
k_{max}	0,0441	0,0234	0,0144	0,0358
Jet-structure coeff. a	0,115	0,084	0,0788	0,0767
Transformed longitudinal coordinate $2ax/d_r$	29,1	21,3	19,6	14,4

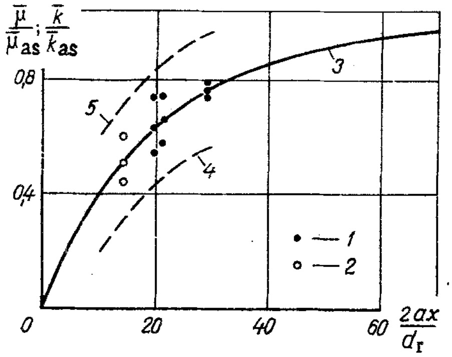

Fig. 1

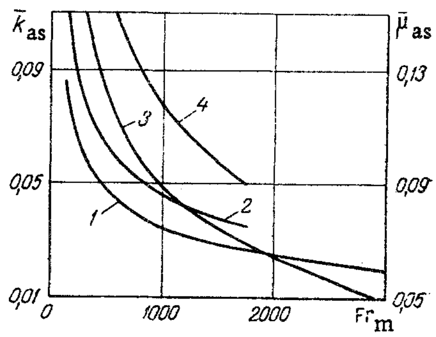

Fig. 2

Fig. 1. Relative mixing coefficient as a function of transformed longitudinal coordinate: 1) data of [2]; 2) data of [3]; 3) from (26); 4 and 5) curves characterizing the limiting deviations of the coefficient.

Fig. 2. The Fr_m dependence of the asymptotic values of the mixing coefficient: 1) $\overline{k}_{as}$ (Na) (27); 2) $\overline{k}_{as}$ (air) (28); 3) $\overline{\mu}_{as}$, 1/cm (Na) (29); 4) $\overline{\mu}_{as}$, 1/cm (air) (30).

and the data on $\overline{k}$ given by (16) and in Table 1 are closely described by

$$\frac{\overline{k}}{\overline{k}_{as}} = 1 - \exp\left[-0.0504\left(\frac{2ax}{d_r}\right)\right] \tag{26}$$

provided that the following is obeyed for sodium:

$$\overline{k}_{as} = \frac{0,77}{Fr_m^{0,451}} \tag{27}$$

and the following for air:

$$\overline{k}_{as} = \frac{1.02}{Fr_m^{0,451}} \tag{28}$$

Figure 1 shows the $\bar{k}$ calculated from (16) for the experimental data [2, 3] together with the relationship of (26); the agreement is good. Figure 2 shows curves from (27) and (28), as well as the results for $\bar{\mu}_{as}$, which take the following forms, respectively, for sodium and air:

$$\bar{\mu}_{as} = \frac{4d_e}{p^2} \frac{0.77}{Fr_m^{0.451}}, \quad (29)$$

$$\bar{\mu}_{as} = \frac{4d_e}{p^2} \frac{1.02}{Fr_m^{0.451}}. \quad (30)$$

The $\bar{\mu}/\bar{\mu}_{as}$ relation is identical to (26) (Fig. 1); Figs. 1 and 2 show that the experimental data of [2, 3] for $\bar{\mu}$ lie somewhat below $\bar{\mu}_{as}$, and they are the lower, the larger Fr_m for a given bundle length.

NOTATION

G_{ij}, transverse flow from cell i into cell j per unit length; G_i, axial flow rate; T, temperature; u, velocity; ρ, density; P, pressure; x, r, coordinates; ξ, hydraulic-resistance coefficient; Pr_T, turbulent Prandtl number; c_p, specific heat; q_V, volumetric heat-production rate; λ, thermal conductivity; R_0, bundle radius; F_u, useful cross section; Π_w, wetted perimeter; μ, viscosity; Re, Reynolds number; Fr_m, centrifugal flow factor.

LITERATURE CITED

1. V. I. Subbotin, P. A. Ushakov, A. V. Zhukov, and V. D. Talakov, "Temperature patterns in fuel elements with liquid-metal cooling," At. Energ., 22, No. 5 (1967).
2. W. Baumann and H. Hoffman, "Coolant cross-mixing of sodium flowing in line through multirod bundles with different spacer arrangements," Paper presented at 1976 International Heat Transfer Seminar, Trogir, Yugoslavia, September, 1976.
3. W. Baumann and R. Moller, "Experimental study of coolant cross-mixing in multirod bundles, consisting of unfinned, one, three, and six fin fuel rods," ATKE, No. 14, 298-304 (1969).
4. A. V. Zhukov, A. B. Muzhanov, A. P. Sorokin, P. A. Titov, and P. A. Ushakov, "Channel interaction of coolant in bundles of cylindrical rods," Preprint No. 413, FÉI, Obninsk (1973).
5. Ya. V. Shevelev and A. Ya. Kramerov, Engineering Calculations on Nuclear Reactors [in Russian], Atomizdat, Moscow (1964).
6. V. M. Ievlev, "Aspects of the hydrodynamic theory of heat transfer for incompressible fluids," Dokl. Akad. Nauk SSSR, 86, No. 6 (1952).
7. É. I. Molodykh and M. D. Segal', "Calculation of the temperature distribution in a bundle of heat-generating rods," Teplofiz. Vys. Temp., 15, No. 4 (1977).
8. M. D. Segal', "Calculation of the temperature distribution in r, φ, z Geometry for a porous medium with spatially uneven heat production," Preprint No. 2845, IAE im. Kurchatova (1977).
9. L. I. Sedov, Methods of Similarity and Dimensions in Mechanics [in Russian], GITTL, Moscow (1954).
10. G. N. Abramovich (editor), Turbulent Jets of Air, Plasma, and Real Gas, Plenum Publ. (1969).